Rational Functions / Algebraic Fractions

When adding or simplifying fractions it helps to have denominators in a fully-factorised form.

Jun 2012, Q1

Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

Difference of two squares

(4)

① Fully factorise the denominators and cancel where possible

$$\frac{2(3x+2)}{(3x+2)(3x-2)} - \frac{2}{3x+1} = \frac{\cancel{(3x+2)}}{\cancel{(3x+2)}} \times \frac{2}{3x-2} - \frac{2}{3x+1}$$

"Cancellation means has the same factor on top and bottom"

$$= \frac{2}{3x-2} - \frac{2}{3x+1}$$

② Using the least number of multiplications possible, multiply by 1 in the form $\frac{f(x)}{f(x)}$ to get all denominators the same.

$$\frac{2}{(3x-2)} \times \frac{3x+1}{3x+1} - \frac{2}{3x+1} \times \frac{3x-2}{3x-2}$$

$$= \frac{\cancel{6x} + 2 - \cancel{6x} + 4}{(3x-2)(3x+1)} = \frac{6}{\underbrace{(3x-2)(3x+1)}}$$

Do not multiply fully factorised denominators

Q1, (Jun 2010, Q1)

Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

$$= \frac{(2x-1)(x+5)}{(x+5)(x-3)} = \cancel{\frac{(x+5)}{(x+5)}} \times \frac{(2x-1)}{(x-3)} = \frac{2x-1}{x-3}$$

Q2, (Jun 2006, Q1)

(a) Simplify $\dfrac{3x^2-x-2}{x^2-1}$.

(3)

(b) Hence, or otherwise, express $\dfrac{3x^2-x-2}{x^2-1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

a/ $$\frac{(3x+2)(x-1)}{(x+1)(x-1)} = \cancel{\frac{x-1}{x-1}} \times \frac{3x+2}{x+1} = \frac{3x+2}{x+1}$$

b/ $$\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{3x+2}{x+1} \times \frac{x}{x} - \frac{1}{x(x-1)}$$

$$= \frac{3x^2+2x-1}{x(x+1)} = \frac{(3x-1)(x+1)}{x(x+1)}$$

$$= \frac{x+1}{x+1} \times \frac{3x-1}{x} = \boxed{\frac{3x-1}{x}}$$

Q3, (Jun 2007, Q2)

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2},$$

Show that $f(x) = \dfrac{4x-6}{2x-1}$.

(7)

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{(2x-1)(x+2)} = \frac{2x+3}{x+2} \times \frac{2x-1}{2x-1} - \frac{9+2x}{(2x-1)(x+2)}$$

$$= \frac{4x^2+4x-3-9-2x}{(2x-1)(x+2)} = \frac{4x^2+2x-12}{(2x-1)(x+2)}$$

$$= \frac{(4x-6)\cancel{(x+2)}}{(2x-1)\cancel{(x+2)}} = \frac{4x-6}{2x-1}$$

Q4, (Jan 2009, Q2)

$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

Express f(x) as a single fraction in its simplest form.

(4)

$$\frac{2x+2}{(x-3)(x+1)} - \frac{x+1}{x-3} = \frac{2\cancel{(x+1)}}{(x-3)\cancel{(x+1)}} - \frac{x+1}{x-3}$$

$$= \frac{2-x-1}{x-3} = \frac{1-x}{x-3}$$

Q5, (Jun 2009, Q7)

The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$$

Show that $f(x) = \frac{x-3}{x-2}$

(5)

$$= 1 \times \frac{(x-2)(x+4)}{(x-2)(x+4)} - \frac{2}{x+4} \times \frac{x-2}{x-2} + \frac{x-8}{(x-2)(x+4)}$$

$$= \frac{x^2+\cancel{2x}-8-(\cancel{2x}-4)+x-8}{(x-2)(x+4)} = \frac{x^2+x-12}{(x-2)(x+4)}$$

$$= \frac{\cancel{(x+4)}(x-3)}{(x-2)\cancel{(x+4)}} = \frac{x-3}{x-2}$$

Q6, (Jan 2010, Q1)

Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

$$= \frac{x+1}{3(x^2-1)} - \frac{1}{3x+1} = \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$$

$$= \cancel{\frac{x+1}{x+1}} \times \frac{1}{3(x-1)} - \frac{1}{3x+1}$$

$$= \frac{1}{3(x-1)} \times \frac{3x+1}{3x+1} - \frac{1}{3x+1} \times \frac{3(x-1)}{3(x-1)}$$

$$= \frac{\cancel{3x}+1-(\cancel{3x}-3)}{3(x-1)(3x+1)} = \frac{4}{3(x-1)(3x+1)}$$

Q7, (Jan 2006, Q2)

Express

$$\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{x^2-x-2}$$

as a single fraction in its simplest form.

(7)

$$= \frac{x\cancel{(2x+3)}}{\cancel{(2x+3)}(x-2)} - \frac{6}{(x+1)(x-2)}$$

$$= \frac{x}{x-2} \times \frac{x+1}{x+1} - \frac{6}{(x+1)(x-2)} = \frac{x^2+x-6}{(x+1)(x-2)}$$

$$= \frac{(x+3)\cancel{(x-2)}}{(x+1)\cancel{(x-2)}} = \frac{x+3}{x+1}$$

Q8, (Jan 2011, Q2)

(a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

a/ $= \frac{4x-1}{2(x-1)} \times \frac{(2x-1)}{(2x-1)} - \frac{3}{2(x-1)(2x-1)}$

$= \frac{8x^2 - 4x - 2x + 1 - 3}{2(x-1)(2x-1)} = \frac{8x^2 - 6x - 2}{2(x-1)(2x-1)}$

$= \frac{\cancel{2}(4x^2 - 3x - 1)}{\cancel{2}(x-1)(2x-1)} = \frac{(4x+1)\cancel{(x-1)}}{\cancel{(x-1)}(2x-1)}$

$= \frac{4x+1}{2x-1}$

b/ $f(x) = \frac{4x+1}{2x-1} - 2 \times \frac{(2x-1)}{(2x-1)} = \frac{\cancel{4x}+1-(\cancel{4x}-2)}{2x-1}$

$= \frac{3}{2x-1}$

Q9, (Jun 2011, Q7)

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9},$$

Show that

$$f(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

$$f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x-3)(x+3)}$$

$$= \frac{4x-5}{(2x+1)(x-3)} \times \frac{x+3}{x+3} - \frac{2x}{(x-3)(x+3)} \times \frac{2x+1}{2x+1}$$

$$= \frac{\cancel{4x^2} + 12x - 5x - 15 - (\cancel{4x^2} + 2x)}{(2x+1)(x+3)(x-3)}$$

$$= \frac{5x-15}{(2x+1)(x+3)(x-3)} = \frac{5\cancel{(x-3)}}{(2x+1)(x+3)\cancel{(x-3)}} = \frac{5}{(2x+1)(x+3)}$$